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Modelling the uncertainty about crop prices and yields using intervals: The minmax regret approach

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Abstract: Agenda 2000, the reform of the CAP (Common Agricultural Policy) in progress, implies significant mutations in the technical and economical environment of French farms. Therefore, an important question for the civil authorities is: "What would be the impact of such changes on cultivated surfaces?" To this end, a micro-economic model, whose purpose was to study the regional agricultural supply, was built by Sourie *et al.* [12] at INRA-ESR (National Institute of Agricultural Research, Rural Economy and Sociology, Grignon, France). This model, named MAORIE - the acronym in French for "Regional Agricultural Supply Model INRA Economy" - is a Linear Programming (LP) model intended to represent the farmers' behavior as to their surface allocations to various cultures. The aim of the present study is to improve the representativity of MAORIE by taking into account uncertainty about crop prices and yields. To model this uncertainty, intervals on gross margins per surface unit were introduced in the objective function level of the model. The resulting model is an "Interval Linear Programming (ILP)" model. Many resolution procedures have been proposed for ILP models in the literature. In this work, the minmax regret approach will be investigated to determine whether it can lead to the desired representativity.

Keywords: Interval Programming, Multiobjective Linear Programming, MinMax Regret, Uncertainty, Behavior Estimation, Agricultural Policies.

1. Introduction

The political and economical orientations of the European Union for the years 2000-2006 were determined on March 1999 by the Berlin Agreements. The new "Common Agricultural Policy (CAP)" adopted for the European Union countries imposes significant changes in the technical and economical environment of European farmers. The main objective is to level European prices with the world market prices and, therefore, increase the competitiveness of the Union in agricultural markets. For French farms, these mutations imply an important reduction in prices and in the amount of subsidies allocated for oilseeds, cereals and protein seeds.

A natural concern of the French Government is, therefore, the evolution of the surfaces allocated to these crops and the income of the farmers. To analyze the possible evolutions, the Rural Economy and Sociology team of the French National Institute of Agricultural Research (INRA-ESR) built MAORIE, a Linear Programming (LP) model [12]. This model aggregates elementary LP models in which each model represents a farm. The purpose of the model is to anticipate farmers' behavior concerning the surfaces allocated for various crops. There are two distinct stages in the exploitation of the model. The first stage is a preparation phase where the parameters of the model are adjusted to the set of farms under consideration until the model becomes able to reproduce "the observed surface allocations" (referred to also as observed solution or observed behavior in the text) for current prices and yields. The second stage is a simulation phase where the future situation is *explored* for different scenarios on future prices (see Figure 1).

The model was implemented by the INRA-ESR team for thirty elementary models corresponding to thirty farms of the Poitou-Charentes[1] Region in France. The results of the first stage showed that the model could not satisfactorily reproduce the observed behavior of the farmers. This was possibly due to the combined effects of the uncertainty about crop prices and yields, two major components of the unit profits which are supposed to guide the choices of farmers. The aim of the present work is to investigate if the representativity of MAORIE concerning farmers' behavior can be improved by taking into account price and yield variations. To model this uncertainty, intervals on gross margins per surface unit were introduced into the objective function of the model. In particular, by means of experiments, an attempt was made to see if it is reasonable to represent farmers' behaviors using the minmax regret approach.

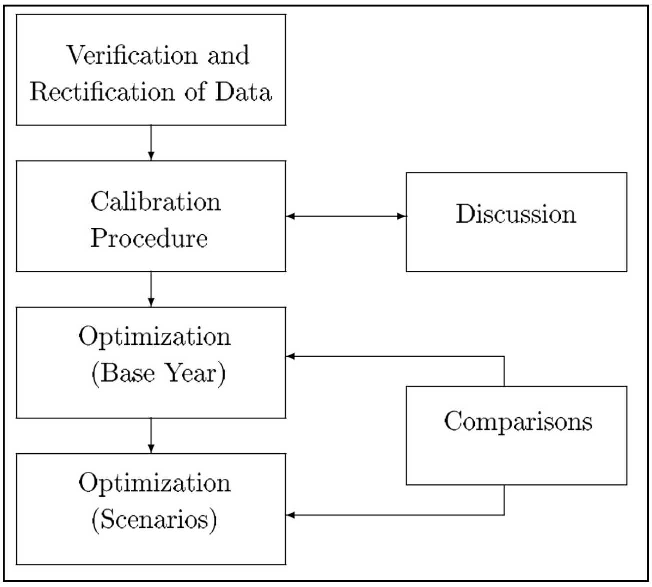

Figure 1. Preparation and Exploitation of MAORIE.

The paper is organized as follows: The necessary information on the mathematical structure of the MAORIE model is given in the next section (more detailed presentations and discussions about the structure of MAORIE can be found in Sourie *et al.* [12] and Kazakci [5]). Formal aspects of the "Interval Linear Programming (ILP)" approach are summarized in section 3. The use of the minmax regret criterion within the ILP framework is then presented in section 4. The implementation procedure and the results thereof are the focus points of section 5. Finally, the conclusions drawn are given in section 6.

2. Modelling the Farmers' Behavior: The MAORIE Model

Structure MAORIE is a linear programming model, which is itself an aggregate of several elementary independent linear programming models. This aggregation (in this context, the summation of elementary models) is possible because there is no common constraint between elementary models[2] each corresponding to one of the farms of the department or the region under consideration.

[1] This region is particularly vulnerable to the changes to come since it ranks fourth in cereal production, first in sunflower production (% 25 of the total French production), and third in corn grain production in France.

[2] Remark, however, that we shall use the elementary models forming MAORIE and not their aggregation since the computational complexity of the adopted methods increases exponentially with the number of decision variables with interval coefficients.

Variables The variables of each elementary model represent surfaces (ha) to be allocated to the production of various crops by the corresponding farm. Thus, they are real and positive. For each farm, the number of variables varies between 8 and 10. The total number of crops considered in MAORIE is 13.

Objective Function The model determines the surface allocation for each elementary model by maximizing the total gross margin of that farm (while respecting certain constraints on the allocations and variations of surfaces). Hence, the objective function for the f^{th} elementary model is:

$$z^f = \sum_{i \in I} \left(p_i \cdot y_{i,f} + s_i - ch_{i,f} \right) \cdot x_{i,f}$$

where $i \in I$ is the index for crops, p_i is the price for the i^{th} crop, $y_{i,f}$ is the yield for the i^{th} crop on the f^{th} farm, s_i is the subsidy for the i^{th} crop, $ch_{i,f}$ is the production cost for the i^{th} crop on the f^{th} farm, and finally, $x_{i,f}$ is the surface allocated for the i^{th} crop by the f^{th} farm. The "*optimal*" solution of MAORIE is simply the aggregate of the optimal solutions of each elementary model: the total surface allocated to a crop at the macroscopic level (when all the exploitations are considered) is equal to the sum of the surfaces allocated to this same crop in the optimal solutions of each elementary model.

Constraints There exist several types of constraints in MAORIE: Land resource constraints, set aside constraints, quotas on demand, irrigation constraints, etc. They can be categorized into the two following groups.

Explicit Agronomic Constraints. These constraints are explicit in the sense that it is easy to interpret their meaning. Land resource constraints for each farm limit the total arable land to its observed value, set aside constraints translate the administrative obligations imposed by the revised Common Agricultural Policy of the E.U. for the set aside land, irrigation constraints give the observed upper bounds for the irrigable surface. Their parameters are easily determined by historical data and observation.

Implicit Agronomic Constraints. These constraints (also referred to as flexibility constraints) give upper bounds on surfaces for crops or groups of crops. They are implicit in the sense that they represent implicitly other constraints (such as availability of labor, technical and technological means,financial resources, etc.) that are not directly represented in the model due to modelling difficulties. For each farm, these upper bounds are a fraction of the total available land for that farm. These fractions are determined for crops and groups of crops and not for the farms. Hence, the same fraction applies to all of the farms considered once determined. The initial values for these fractions are determined by observing the historical data (see Kazakci [5] for details). Then, a trial-error process, called the calibration procedure, is undertaken: the model is solved, the returned solution is compared to the observed solution to see the *distance*. If the distance is important, a new set of parameters for the flexibility constraints is determined, and the process continues until a satisfactory solution is obtained.

For the purpose of this paper, the fth elementary model of MAORIE can be represented by the following LP model.

$\max\{z^f = \mathbf{c}^f \mathbf{x}^f : \mathbf{x}^f \in S^f\}$

where $\mathbf{c}^f$ is the vector of unit gross margins for the f^{th} farm, $\mathbf{x}^f$ is the decision vector and S^f represents the feasible decisions for the f^{th} farm.

Results of the Calibration Procedure for Poitou-Charentes Figure 2 presents the aggregated observed solution for the considered region and the MAORIE results (i.e., the sum of optimal allocations by farm for each crop) at the end of the calibration procedure. The surfaces are expressed in hectares. As one can see, there exists some considerable gaps between the observed and the optimized allocations for various crops: rape-seed, irrigated and non-irrigated corn, sunflower, barley(food) and wheat. The difference in absolute value between the observed production levels and the optimized allocations (in other words, the distance between the two solutions using a L1 metric) is 1624 ha. The total arable land for the region being 4282 ha, the relative distance (the difference between the two solutions in absolute value divided by the total arable land) is 38%. In fact, at the microscopic level (i.e. when the results of elementary models are considered one by one), the distances become more important: for 26 farms out of 30, the relative distance is more than 40%, for 20 farms out of 30, the distance is even higher (50%). At the microscopic level, these results could be expected, since by summing the allocation levels we introduce compensatory effects (see Kazakci [5] for a detailed analysis).

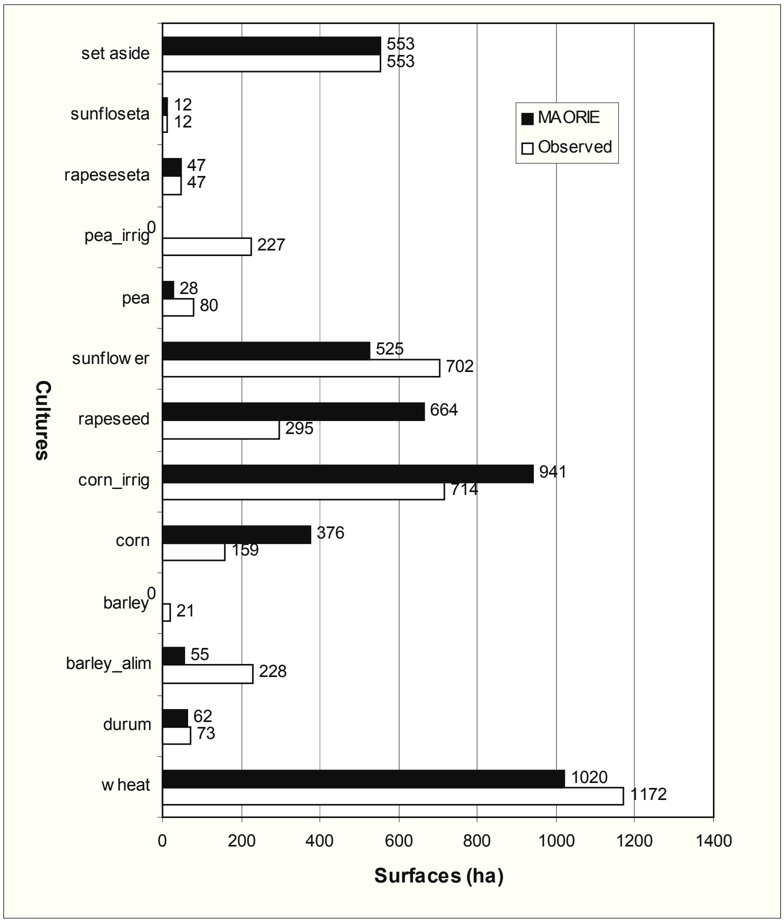

Figure 2. Comparison of the Observed and the Optimized (MAORIE) Solutions at the Macroscopic Level.

Hence, the need for improvement in the representativity of the model is clear. In all evidence, such distances can occur for two reasons: an inaccurate specification of the feasible regions of the models (which would be closely related to the calibration procedure) or an inaccurate specification of the objective functions. Considering the recent changes in the economical environment and the natural uncertainty of the yields, we opted for investigating the problems that may arise because of a possibly inaccurate specification of the objective functions[3]. More precisely, we modified the original MAORIE model to take into account uncertainty about prices and yields by using interval valued coefficients for the objective function. The presentation of the formal interval linear programming problem is given in the next section.

3. Interval Linear Programming

In order to decide about their surface allocations, the farmers have to estimate future crop prices and yields. In a relatively stable environment, it is reasonable to suppose that farmers will base their decisions on average prices and yields. MAORIE is originally designed under this very assumption: objective function coefficients (the gross margins per crop) are calculated based on the 1993-1997 price and yield averages.

In the present context, however, the unpredictability of the unit gains increases: the natural uncertainty about yields is combined with an exceptional uncertainty about prices. Therefore, the farmers would be less willing to use average prices and yields to make their decisions. Instead, a natural tendency for them would be to base their reasoning on the ranges of variation in prices and yields. For this reason, the objective function coefficients, which correspond to unit gross margins per crop, will be represented by intervals in the modelling. In the following paragraph, a brief review of the literature devoted to the subject and a formal definition of the Interval Linear Programming (ILP) problem will be given. Finally, two kinds of approaches concerning possible solution procedures will be outlined.

3.1 Related Work on ILP Models

In mathematical programming models, the coefficient values are often considered known and fixed in a deterministic way. However, in practical situations, these values are frequently unknown or difficult to determine precisely.

Interval Programming (IP) has been proposed as a means of avoiding the resulting modelling difficulties, by proceeding only with simple information on the variation range of the coefficients. More precisely, it consists of using parameters whose values can vary within some interval, instead of parameters with fixed values, as is the case in conventional mathematical programming. Many techniques have been proposed to solve the resulting problem. To our knowledge, the proposed techniques consider only linear mathematical programming models.

Shaocheng [10] studied the case where all the model parameters are represented by intervals and the decision variables are non negative. Recently, Chinneck and Ramadan [2] generalized their approach to the case where variables are without sign restriction. The case which is of greater interest for our purpose is the one where only the objective function coefficients are represented by intervals. This particular problem is the most frequently considered in ILP literature (see, e.g., [1], [3], [4], [6], [8], [7], [9], [13]). We now introduce some definitions and notations and briefly present the formal problem.

3.2 Interval Linear Programming (ILP) Problem

Let us consider a Linear Programming (LP) model with n (real and positive) variables and m constraints. The objective function is to be maximized. Formally:

[3] Therefore, an implicit hypothesis is that the feasible region of each elementary model represents adequately the allocation possibilities of the farmers. Let us note that the observed solutions for each farm have been verified to be feasible in the corresponding model.

max {**cx** : c ∈ Γ, **x** ∈ S} (ILP)

where

$$\Gamma = \{c \in \Re^n : c_i \in [l_i, u_i], \forall i = 1..n\}$$

$$S = \{x \in \Re^n : Ax \le b, x \ge 0, A \in \Re^{m \times n}, b \in \Re^m\}$$

The uppercase letters with bold characters denote matrices (e.g., **A**). The lowercase letters with bold characters denote vectors (e.g., $c \in \Re^n, x \in \Re^n$). The null vector is denoted by 0. [l_i, u_i] represents a closed interval of real numbers where l_i stands for the lower bound and u_i stands for the upper bound. The letters with indices indicate

the elements of a matrix (e.g., $A = (a_{ij})_{m,n} \in \Re^{m \times n}$) or a vector (e.g., $c = (c_1, ..., c_i, ..., c_n) \in \Re^n$) or an interval (e.g., $l_i \in [l_i, u_i]$).

Let $\Pi = \{x \in S : x = \arg\max\{cy : y \in S, c \in \Gamma\}\}$ be the set of potentially optimal solutions. Let Υ be the set of all the extreme objective functions: $\Upsilon = \{c \in \Gamma : c_i \in [l_i, u_i], \forall i = 1..n\}$.To give insight into what the problem becomes when intervals are introduced, we recall the following theorem [13], [3]:

Theorem 1

Let us consider the following multiobjective linear programming problem:

*v–max{**cx** : **x** ∈ S; c ∈ Υ}* *(MOLP)*

where the v-max notation stands for the vector maximization. Then, a solution is a potentially optimal solution to (ILP) problem if, and only if, it is weakly efficient to the (MOLP) problem.

Hence, (ILP) is a particular multiobjective linear programming problem where the 2^n objectives are elements of Υ and the set of potentially optimal solutions Π is the set of weakly efficient solutions to (MOLP). Theoretically, this knowledge enables us to mobilize all the tools and concepts of multiobjective linear programming literature, especially to choose/propose suitable solution concepts for (ILP) problem. In the literature, two distinct attitudes can be observed. The first attitude consists of finding all potentially optimal solutions that the model can return in order to examine the possible evolutions of the system that the model is representing. The methods proposed by Steuer [13] and Bitran [1] follow this kind of logic. The second attitude consists of adopting a specific criterion (such as the Hurwicz's criterion, the maxmin gain of Falk, the minmax regret of Savage, etc.) to select a solution among the potentially optimal solutions. Rommelfanger et al. [9], Ishibuchi and Tanaka [4], Inuiguchi and Sakawa [3] and Mausser and Laguna [6], [7], [8] proposed different methods with this second perspective. Following this perspective, the next section introduces the approach that we have selected, namely the minimization of the maximum regret approach, and the procedure we adopted for its implementation.

4. Minimizing the Maximum Regret

Minimizing the maximum regret consists of finding a solution which will give the decision maker a satisfaction level as close as possible to the optimal situation (which can only be known as a *posteriori*), whatever situation occurs in the future. The farmers of the Poitou-Charentes region are faced with a highly unstable economic situation and know that their decisions will be based on uncertain gains. It seems reasonable to suppose that they will decide on their surface allocations *prudently* in order to go through this time of economic unstability with minimum loss, while trying to obtain a satisfying profit level. This is precisely the logic underlying the minmax regret criterion; i.e. selection of a *robust* solution that will give a high satisfaction level whatever happens in the future and that will not cause regret. Therefore, we make the hypothesis that the farmers of the considered region adopt the min-max regret criterion to make their surface allocation decisions. The mathematical translation of this hypothesis for the MAORIE was to implement the minmax regret solution procedure proposed in the literature (see e.g., [3], [6], [7], [8]). The presentation of the formal problem and the algorithm of minmax regret are presented in the next paragraphs.

4.1 MinMax Regret (MMR) Problem

Suppose that a solution $x \in S$ is selected for a given $c \in \Gamma$. The regret is then:

$$R(c,x) = \max_{y \in S} \{cy\} - cx$$

The maximum regret is:

$$\max_{c \in \Gamma} \{R(c,x)\}$$

The *minmax* regret solution $\hat{x}$ is then such that $R_{\max}(\hat{x}) \leq R_{\max}(x)$ for all $x \in S$.
The corresponding problem to be solved is:

$$\min_{x \in S} \{\max_{c \in \Gamma} \{\max_{y \in S} \{cy\} - cx\}\} \qquad \textit{(MMR)}$$

4.2 The MinMax Regret Algorithm

The main difficulty in solving (*MMR*) lies in to the infinity of objective functions to be considered. Shimizu and Aiyoshi [11] proposed a relaxation procedure to handle this problem. Instead of considering all possible objective functions, they consider only a limited number among them and solve a relaxed problem (hereafter called (*MMR'*)) to obtain a candidate regret solution. A second problem (called hereafter (*CMR*)) is then solved to test the global optimality of the generated solution. If the solution is globally optimal, the algorithm terminates. Otherwise, (*CMR*) generates a constraint which is then integrated into the constraint system of (*MMR'*) to solve it again for a new candidate solution. This process continues in this manner until a globally optimal solution is obtained. The relaxed (*MMR'*) problem is:

$$\min_{x \in S} \{\max_{c \in \Gamma} \{\max_{y \in S} \{cy\} - cx\}\} \qquad (\textit{MMR'})$$

where $C = \{c^1, c^2, \ldots, c^p\} \subset \Gamma$. This problem is equivalent to:

$$\min r \qquad (\textit{MMR'})$$

s.t. $r + c^k x \geq c^k x_{c^k}$, $k = 1, \ldots, p$

$r \geq 0,\ x \in S,\ c^k \in C$

where x_{c^k} is the optimal solution of $\max_{y \in S, c^k \in C}(c^k y)$. A constraint of type $r + c^k x \geq c^k x_{c^k}$ is called a regret cut. Let us denote $\bar{x}$ the optimal solution of (*MMR'*) and $\bar{r}$ the corresponding regret. Since all possible objective functions are not considered in (*MMR'*) we cannot be sure that there is no c belonging to $\Gamma \setminus C$ which can cause a greater regret by its realization in the future. Hence, we use the following (CMR) problem to test the global optimality of $\bar{x}$:

$$\max_{c\in\Gamma}\left\{\max_{y\in S}\{cy\}-c\bar{x}\right\} \qquad (CMR)$$

Observe that the objective function value of (CMR) represents the maximum regret for $\bar{x}$ over Γ, denoted by $R_{\max}(\bar{x})$. If the optimal solution $x_{c^{p+1}} \in S, c^{p+1} \in \Gamma$ of (CMR) gives $R_{\max}(\bar{x}) > \bar{r}$, it means that c^{p+1} can cause a greater regret than $\bar{r}$ by its realization in the future and that it has to be considered also in C while solving (*MMR'*). So, the regret cut $r + c^{p+1}x \geq c^{p+1}x_{c^{p+1}}$ is added to the previous constraint set of the (*MMR')* to solve it again and obtain a new candidate. The process is iterated until the generated candidate regret solution is found to be optimal by (CMR). This solution procedure idea is summarized with the following algorithm:

MinMax Regret Algorithm

Step 0: $r^\circ \leftarrow 0, k \leftarrow 0,$ choose an initial candidate $\bar{x}$

Step 1: $k \leftarrow k+1,$ Solve (CMR) to find c^k and $R_{\max}(\bar{x})$:

If $R_{\max}(\bar{x}) = r^\circ$ then END. $\bar{x}$ minimize the maximum regret.

Step 2: Add the regret cut $r + c^k x \geq c^k x_{c^k}$ to the constraint set of (*MMR'*)

Step 3: Solve (*MMR'*) to obtain a new candidate $\bar{x}$ and $\bar{r}$. $r^\circ \leftarrow \bar{r}$. Go to Step 1.

The difficulty in this resolution process lies in the quadratic nature of the (CMR) problem. Inuiguchi and Sakawa [3] investigated the properties of the minmax regret solution to find a more suitable way to solve (CRM). Mausser and Laguna [6] used their results to formulate a mixed integer linear program equivalent to (CMR) which is less costly to solve. In our experiments we used this equivalent problem formulation.

5. Implementation of the MMR Approach

We implemented the interval linear programming approach with the minmax regret criterion to the MAORIE model to investigate if its representativity can be improved by this approach. The thirty elementary models corresponding to the farms of Poitou-Charentes region were considered. Each model had between 8-10 variables. The total number of crops considered was 13. The three variables representing set aside, set aside for rape-seed and set aside for sunflower were common to all the models. These crops are not very interesting economically, and their part in the total arable land is imposed by the government. The economic instability does not affect their practice. For these reasons, we did not use interval coefficients for them and kept the original parameters (gross margins) of MAORIE. Hence, for our (ILP) models, the number s of interval valued coefficients varied between 5 and 7 (the 5-7 first variables in the models).

An important point in our experiments was the choice of the intervals for the objective function coefficients. We tested 5 various sets of interval coefficients: for the j^{th} implementation, the intervals for coefficients were obtained by $\left[l_i = (1-0.1j)c_i, u_i = (1-0.1j)c_i\right]$ where (*i = 1, …, s*) were the original coefficients (gross margins) used in the MAORIE model. Remark that, as the experiment index *j* goes from 1 to 5, the intervals get larger and the possibility that the various crops have intersecting interval gain increases.

We used the GAMS software to implement the given minmax regret algorithm and the linear and integer programming modules of the CPLEX solver. For the initial regret candidates to start the algorithm, we used the optimal solutions of MAORIE.

To evaluate the proximity of the j^{th} minmax regret solution x_k^j to the observed solution x_k^{obs} for the farm k, we used the following performance measure:

$$M_1^j\left(x^j\right)=\frac{L_1\left(x^j, x^{obs}\right)}{TotalLand}=\frac{\sum_i \left|x_i^j - x_i^{obs}\right|}{\sum_i x_i^{obs}}$$

The results are recapitulated in Figures 3 and 4.

5.1 Analysis of the Results

Let us note two effects of the penny-switching nature of conventional LP in order to better understand the utility of the MMR approach:

Observation 1 Since the gross margin of irrigated peas is in all cases inferior to irrigated corn, MAORIE allocates all the irrigable surfaces to this second crop, although the production of the first one has been observed in 20 out of 30 farms.

Observation 2 For the cases where the gross margin of wheat is inferior to the gross margins of rape-seed and sunflower, no surface was allocated by MAORIE to the wheat although its production was observed in every case.

The principal effect of the ILP approach with the MinMax Regret is:

Observation 3 For the two previous cases, when the differences between the gross margins is relatively small, the minmax regret approach gives more "balanced" solutions, and this more so when the interval coefficients gets larger (i.e., when j increases).

This last observation seems natural. In fact, as the intervals get larger, the interval gains for different crops start to overlap or, if they already have an intersection, they become more overlapping. It becomes more difficult to anticipate which crop will be more profitable. Hence, the minmax regret approach tends to return more and more balanced solutions as the sizes of the intervals increase.

The effects of the minmax regret approach on the proximities obtained at the microscopic level are considerable: for 26 farms out of 30, for all the sets of intervals considered (j = 1, …, 5), the relative distance (M_1^j) of the minmax regret solution to the corresponding observed solution is smaller than the relative distance of the MAORIE's optimum solution to the observed one. Concerning the improvement in the proximities to the observed solutions, the worst proximities ($\max\left(M_1^j\right)$) obtained for these 26 farms provide an average improvement of 11% with respect to MAORIE's proximities (denoted by M_1^j (x^{opt}) in the figures, where x^{opt} stands for the MAORIE's optimal solutions for the corresponding farms). We tried to go deeper in our analysis and reconsidered the initial data with the obtained solutions to extract some possible characterization of the 26 cases where the minmax regret approach improves the representativity of MAORIE, as well as the non improved 4 cases. These four last farms do not share any distinguishable properties (concerning their initial data: gross margins and observed solutions).Therefore, the non improvement with the minmax regret approach is rather due to the particular configuration of the parameter values of the corresponding elementary models. Hence, nothing indicates a possible characterization of these isolated cases. For the other 26 farms, the improvement is mainly due to the fact that, in general, farmers adopted more balanced strategies of allocation than the ones suggested by MAORIE, especially concerning the trio wheat - sunflower - rape-seed and the duo irrigated peas - irrigated corn. From these observations, we may conclude:

Conclusion 1 In the cases where the farmers choose a balanced allocation, the min-max regret solutions tend to improve the representativity of the model, otherwise the proximities to the observed solutions get worse.

Conclusion 2 Farmers' decisions are not based on the maximization of the profit logic underlying the MAORIE model.

Farms	M_1^1	M_1^2	M_1^3	M_1^4	M_1^5	Average M_1j	Min M_1j	Max M_1j	M_1(xopt)
cha17 I26	0,28	0,27	0,30	0,27	0,34	0,29	0,27	0,34	0,34
cha17 I15	0,70	0,54	0,46	0,50	0,48	0,53	0,46	0,70	0,86
2sa79	0,60	0,60	0,41	0,39	0,45	0,49	0,39	0,60	0,69
2sa80	0,88	0,72	0,57	0,57	0,61	0,67	0,57	0,88	0,89
cha5 I5	0,41	0,28	0,27	0,31	0,32	0,32	0,27	0,41	0,41
cha7 I31	0,22	0,22	0,31	0,38	0,39	0,30	0,22	0,39	0,32
cha74 I36	0,29	0,28	0,35	0,40	0,42	0,35	0,28	0,42	0,41
cha86 I38	0,32	0,32	0,20	0,17	0,16	0,24	0,16	0,32	0,57
chmEDO	0,92	0,92	0,81	0,76	0,73	0,83	0,73	0,92	0,92
vie122	0,48	0,37	0,28	0,23	0,20	0,31	0,20	0,48	0,59
vie25	0,44	0,43	0,35	0,30	0,27	0,36	0,27	0,44	0,56
vie37	0,38	0,39	0,36	0,28	0,23	0,33	0,23	0,39	0,40
vie80	0,36	0,42	0,46	0,47	0,45	0,43	0,36	0,47	0,30
2sa102	0,49	0,23	0,16	0,15	0,16	0,24	0,15	0,49	0,54
2sa104	0,87	0,82	0,69	0,63	0,60	0,72	0,60	0,87	0,87
2sa125	0,79	0,78	0,71	0,68	0,65	0,72	0,65	0,79	0,87
2sa128	0,89	0,87	0,76	0,68	0,69	0,78	0,68	0,89	0,90
2sa137	0,42	0,43	0,39	0,42	0,39	0,41	0,39	0,43	0,31
2sa163	0,54	0,50	0,44	0,44	0,43	0,47	0,43	0,54	0,76
2sa165	0,45	0,23	0,27	0,23	0,24	0,28	0,23	0,45	0,79
2sa166	0,57	0,47	0,39	0,36	0,34	0,43	0,34	0,57	0,65
2sa168	0,44	0,44	0,44	0,41	0,39	0,42	0,39	0,44	0,64
2sa17	0,28	0,19	0,16	0,20	0,21	0,21	0,16	0,28	0,59
2sa191	0,68	0,60	0,56	0,57	0,58	0,60	0,56	0,68	0,51
2sa37	0,34	0,31	0,33	0,31	0,23	0,30	0,23	0,34	0,44
2sa44	0,50	0,43	0,42	0,50	0,51	0,47	0,42	0,51	0,64
2sa66	0,22	0,17	0,19	0,24	0,24	0,21	0,17	0,24	0,41
2sa67	0,50	0,31	0,17	0,06	0,08	0,22	0,06	0,50	0,55
2sa71	0,49	0,39	0,39	0,39	0,40	0,41	0,39	0,49	0,77
2sa87	0,26	0,23	0,29	0,35	0,38	0,30	0,23	0,38	0,49

Figure 3. Proximities obtained by the minmax regret approach versus proximities obtained by MAORIE ($M_1(x^{opt})$).

When we consider the proximities at the global level (the sum of the production levels), we have M_1^1 = 36%; M_1^2 = 30%; M_1^3 = 24%; M_1^4 = 26%; M_1^5 = 27% versus M_1^2 = 38% for MAORIE. As we can see, there is some improvement in every case (see Figure 5). The proximities keep improving for j=1, 2, 3, then slightly deteriorate for j=4, 5. For the first two experiments where the dispersions were relatively small (±%10; ±%20), it seems that there is no real benefit to use the minmax regret approach, since the improvements (36%, 30%) are marginal with respect to MAORIE's representativity (38%). For the last three experiments (j=3, 4, 5), the improvements are better (24%, 26%, 27%) and relatively stable.

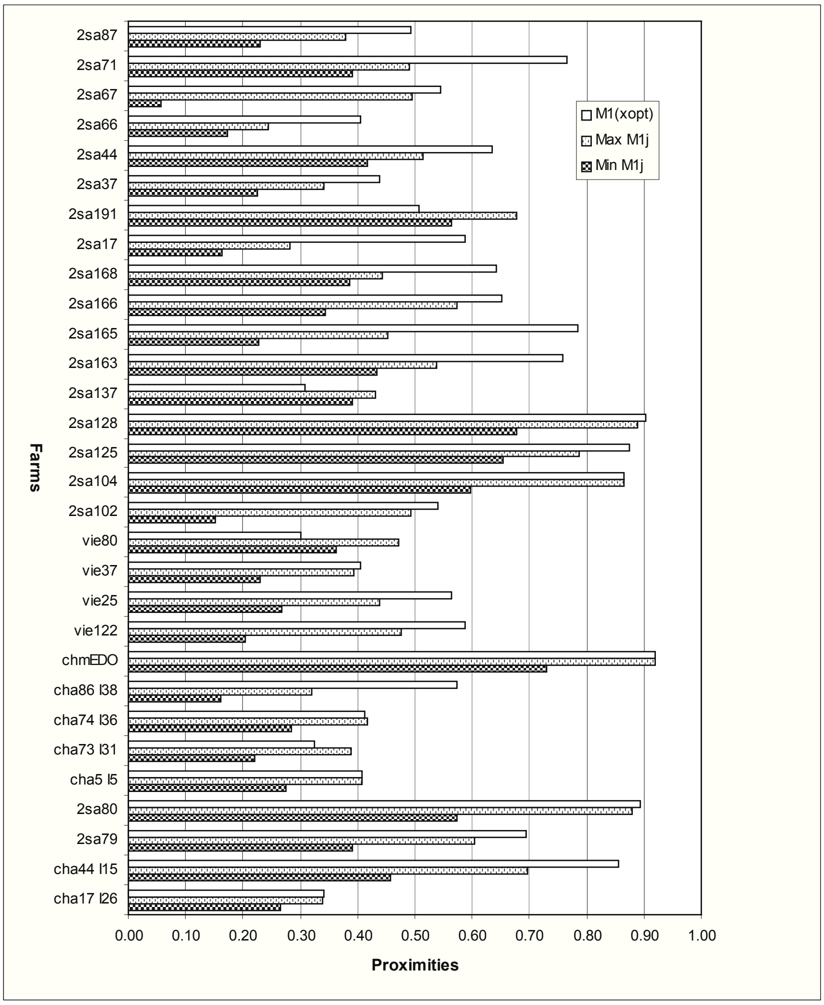

Figure 4. Comparison of the minimum and the maximum proximities obtained by the minmax regret approach versus the proximities obtained by MAORIE.

But these observations hardly mean that our initial hypothesis, i.e. the farmers decide using the minmax regret criterion, is true. Although we obtained some improvements, the best global achievement (M_1^3 = 24%) was still very high. Also, at the microscopic level, there was only one farm for which the proximity to the observed behavior dropped below 10% (and only 5 farms below 20%). Thus, we can conclude the following:

Conclusion 3 The minmax regret approach is not the approach taken by the farmers to decide on their surface allocations, at least not for the sets of intervals we considered. Although some improvement was obtained, our initial hypothesis seems only partially true, since the improvements are hardly satisfactory.

The last conclusion requires that the following questions be answered: Why could further improvements not be obtained? Is this due to the sets of intervals we used? Or is this because the minmax regret criterion does not reflect the farmers' decision criterion? If not, then what constitutes their criterion? Could better solutions be obtained? Using which criterion? These questions are related to the three options we adopted during the study: the acceptance of the feasible regions of the elementary models as adequate, the choice of minmax regret as the criterion to be tested and the set of intervals we used.

It is difficult to predict if some other set of intervals may lead to better proximities. It is possible to undertake a trial and error process, such as the calibration procedure for the MAORIE, to find a more satisfactory set of intervals. Still, in the event of success, it might become difficult to interpret whether the results are merely mathematical or the farmers' behavior has been reproduced.

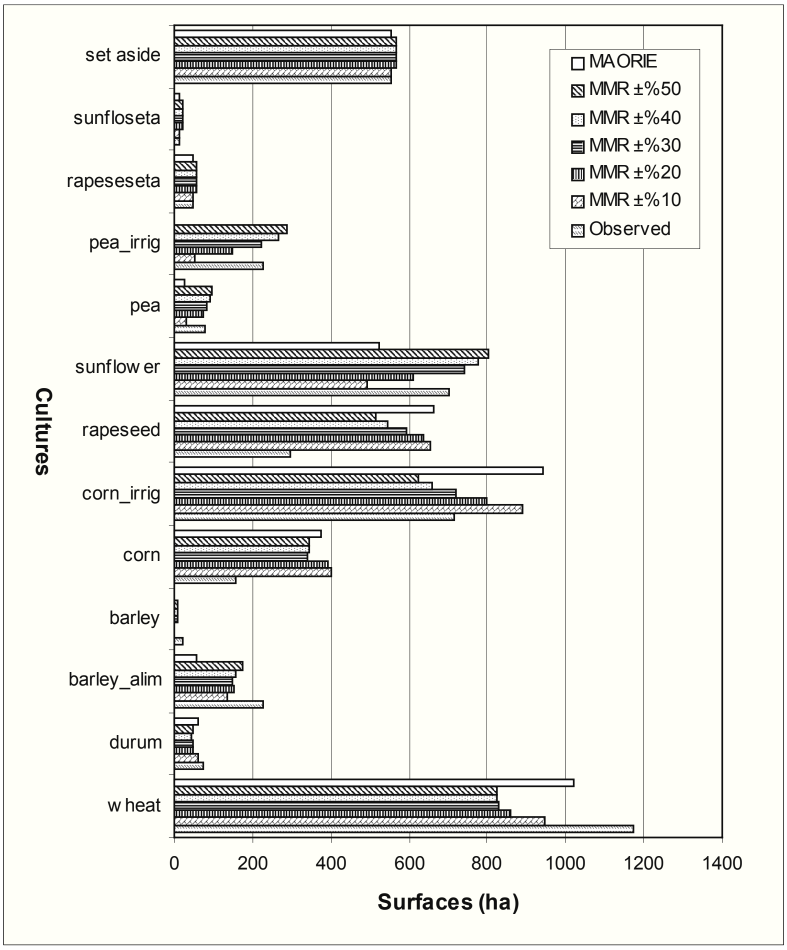

Figure 5.
Comparison of the MMR Solutions with the Observed and the Optimized (MAORIE) Solutions at the Macroscopic Level.

Concerning the criterion we tested, one thing is clear: in reality, farmers have many objectives, such as minimization of the management complexity, minimization of total work time, fulfillment of the conditions of some contract, etc. The results of our experiments indicate that min-max regret better represents the farmers' decisions than the gross margin. However, it is very possible that it does not represent all of the farmers' objectives.

The last point to analyze is the assumption that the feasible regions are adequate. Since we did not obtain a satisfactory proximity, we must ask ourselves about the validity of this assumption. If the non reproduction of the observed behaviors is due to an inaccurate specification of the feasible regions, then this means that the observed solutions are not close to their corresponding sets of potentially optimal solutions. In this case, reconsideration of the feasible regions would be necessary.

Of these three possible sources for the partial improvement of the MAORIE's representativity, we believe that the priority should be given to the investigation of the last one. Hence, the representativity of the feasible regions should be analyzed in order to shed some light on the previous questions.

To evaluate the quality of representation of the feasible regions, information on their structure, especially concerning the sets of potentially optimal solutions corresponding to each elementary model and the positioning of the observed solutions with respect to these sets is necessary. Let us note that, in the context of interval linear programming, such an "information gathering" could neither be done using the minmax regret approach, nor using any of the other methods of the second kind of attitudes (see §3.2). The reason is that these approaches return a unique (or a small number of) solution(s) that cannot give the required information. Thus, we see that methods that use a specific criterion to select a solution for (ILP) models, in spite of their possible utility in a decision-making context, have a limitation in a behavior anticipation context. On the other hand, however, analysis of the structures of the sets of potentially optimal solutions is possible with the first kind of attitude, i.e. by characterizing the sets of potentially optimal solutions or by simply exploring them. Some results on this subject will be reported in a future work.

6. Conclusions

The aim of this study was to improve the representativity of the MAORIE model, a linear programming model intended to represent the behavior of the farmers with respect to their surface allocations to various cultures and to study the impacts of the political changes on cultivated surfaces. The principal goal of this work was to investigate the utility of modelling the uncertainty by interval valued parameters at the objective function level. The resulting model from this approach is called an "interval linear programming model".

Within this framework, we considered thirty elementary linear programming models (forming MAORIE) corresponding to the farms of the Poitou-Charentes area. We hypothesized that farmers' behavior could be better represented using the minmax regret criterion. To test this hypothesis, the Minmax Regret (MMR) algorithm was implemented for each of the thirty models. The aim of the algorithm is to find the solution minimizing the maximum regret for a linear programming model with objective function coe-cients in the form of intervals. Five experiments with five sets of intervals were performed.

Analysis of the results and a comparison with the optimal solutions of MAORIE for the elementary models showed that the MMR approach had a character which softened the sometimes abrupt nature of the linear programming, for which the least difference between the unit profits implies an allocation. On the other hand, the MMR approach gave better balanced and distributed solutions, and this more so when the overlapping of the interval profits for various crops increased.

It was shown that whenever a farmer chose a relatively balanced surface allocation, especially concerning the trio wheat - sunflower - rape-seed and the duo irrigated peas-irrigated corn, the solutions of minmax regret tended to improve the reproduction of the observed solution. We also observed that our hypothesis was only partially true. Although some improvements in the representativity were achieved, the proximities obtained by the MMR approach were not satisfactory enough to support that the farmers decide on their surface allocations according to the logic of minmax regret.

Possible reasons concerning the modelling phase and the working hypotheses, which caused the non-reproduction of the observed behavior, were discussed. We argued that the structures of the sets of potentially optimal solutions of the elementary models should be analyzed to better understand the reasons for unsatisfactory representativity. Some experimental results and comparisons thereof will be discussed in a future work.

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